

# AN 8-GROUP DELAYED NEUTRON MODEL BASED ON A CONSISTENT SET OF HALF-LIVES

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**ABSTRACT** - Using a nonlinear least-squares fitting technique, the group parameters for 245 sets of experimentally-measured delayed neutron group constants for 20 fissionable isotopes have been expanded into an 8-group delayed neutron model based on a consistent set of group half-lives. During the expansion process, the reactivity scale for positive reactivities is conserved, as well as the time-dependent behavior of the reactor system as predicted by the original delayed neutron model. In addition, the mean half-life of the original delayed neutron set is conserved, as well as the overall uncertainty of the reactivity scale as quoted by the original experimenter.

## INTRODUCTION

In April 1997, an international workshop on delayed neutrons was held at the Institute of Physics and Power Engineering (IPPE) in Obninsk, Russia. The workshop was sponsored by the Nuclear Energy Agency's (NEA) working party on delayed neutrons (WPEC/SG6). The primary intent of this workshop was to review the current status of delayed neutron data and to propose new programs to improve these data for applications in reactor physics. Amongst the various proposals, it was suggested that a higher-order delayed neutron model (e.g., 7-, 8-, or 9-group model) be developed in which the group half-lives would be specified at the half-lives of known dominant precursors in each of the various half-life regimes. Most importantly, the half-lives of the three longest-lived groups in the new model would be fixed at the half-lives of the three longest-lived dominant precursors (i.e., <sup>87</sup>Br, <sup>137</sup>I, and <sup>88</sup>Br). It was thought that by specifying these three half-lives, potential biases in the reactivity scale<sup>b</sup> would be reduced when inferring reactivities from period

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<sup>b</sup>. Throughout the remainder of this work, we shall refer to the relationship between period (or inverse period) and reactivity as the *reactivity scale*. This relationship is highly dependent on the value of the delayed neutron parameters assumed in the inhour equation and is very important when measuring reactivity in operating systems.

Table I. Group-1 Delayed Neutron Spectra for  $^{235}\text{U}$  and  $^{238}\text{U}$ .

Hansen-Roach Cross Section Energy Group	$^{235}\text{U}$ Delayed Neutron Spectrum	$^{238}\text{U}$ Delayed Neutron Spectrum
1	0.000	0.000
2	0.004	0.008
3	0.070	0.119
4	0.255	0.364
5	0.483	0.387
6	0.159	0.105
7	0.029	0.017
8-16	0.000	0.000

measurements and/or inverse kinetic calculations.

From a reactor physics standpoint, there were two other reasons that stimulated interest in the development a new delayed neutron model based on a consistent set of half-lives. The first reason was the added attraction of simplifying the dynamic model of complex systems in which more than one fissionable isotope is present. For example, using the current 6-group model, systems containing 5 fissionable isotopes would require 30 differential equations to describe the total delayed neutron activity since each isotope is characterized by its own unique set of half-lives. In comparison, using a delayed neutron model based on, for example, 8 consistent groups, only 8 differential equations would be needed to describe the total delayed neutron activity from all 5 fissionable isotopes.

The second reason for wanting a delayed neutron model based on a consistent set of half-lives was to obtain a more consistent set of delayed neutron spectra. For example, using the current 6-group models for  $^{235}\text{U}$  and  $^{238}\text{U}$ , we note that the spectra for delayed neutron group 1 are quite different (See Table I). In reality, however, the spectra for delayed neutron group 1 should be nearly identical for both  $^{235}\text{U}$  and  $^{238}\text{U}$  since that particular delayed neutron group corresponds predominantly to  $^{87}\text{Br}$ . Under the newly proposed delayed neutron model, the delayed neutron spectrum for group 1 would be nearly identical for all isotopes since the neighboring precursors  $^{137}\text{I}$  and  $^{88}\text{Br}$  would be treated as their own individual delayed neutron group and, as such, would not contribute to the group-1 spectrum.

Fixing the half-lives at specified values is not a new concept. It was first practiced by several experimenters during some of the earlier measurements of delayed neutron group parameters. In particular, Maksyutenko (1958) routinely fixed the half-lives at specified values in order to study the change in the relative abundances of the various groups as a function of incident neutron energy. Keepin (1965) also attempted this idea, but opted to stay with the 6-group formulation in which both the abundances and half-lives were free parameters in the least-squares-fit (LSF) of the delayed neutron decay curve. Years later, Meneley (1970) revisited the issue and suggested that a single set of isotope-independent and energy-independent half-lives

could be determined such that the delayed-neutron activity in composite-fuel cores could be adequately described using six groups. Cahalan and Ott (1973) partially followed Meneley's suggestion and refit the delayed neutron parameters measured by Keepin (1965) to obtain a set of isotope-independent half-lives applicable for fast fission.

In this study, we present results of an 8-group model that uses a consistent set of half-lives that are both isotope-independent and energy-independent. Using a nonlinear least-squares fitting technique, the group parameters for 245 sets of experimentally-measured delayed neutron group constants for 20 fissionable isotopes (Spriggs and Campbell, 1999) have been expanded into an 8-group model. During the expansion process, the reactivity scale for positive reactivities is conserved, as well as conserving the time-dependent behavior of the reactor system as predicted by the original delayed neutron model. In addition, the mean half-life of the original delayed neutron set and the uncertainty of the reactivity scale as quoted by the original experimenter are conserved.

### RATIONALE FOR 8 GROUPS OF DELAYED NEUTRONS

Based on recent studies performed in Russia (Gudkov et al., 1992) and in the United States (Campbell and Spriggs, 1998), it has been shown that at least 82% of all delayed neutrons are produced by a dozen or so precursors that are common to a large number of fissioning isotopes. Therefore, it seems that a viable alternative to the current 6-group model is to increase the number of delayed neutron groups and to fix the half-lives in the new model to some suitable set of values that cover the known range of precursor half-lives.

In particular, it was proposed at the Obninsk workshop that the half-life of group 1 be fixed at the half-life of  $^{87}\text{Br}$  and that group 2 in the current 6-group model be separated into two separate groups—one for  $^{137}\text{I}$  and one for  $^{88}\text{Br}$ . From an experimental standpoint, this separation has already been demonstrated to be feasible by Charlton et al. (1996, 1997, 1998a, 1998b). It was also suggested that the short-lived group (i.e., group 6 of the current 6-group model) be split into two groups in order to cover the wide range of half-lives observed for that group (see Table II). This separation has also been demonstrated to be feasible by Piksaikin (1998) using a periodic irradiation technique that allows for more resolution of the decay curve at short times. As for the remaining interior group half-lives, it was suggested that they be fixed at the half-life of either a dominant precursor or at some average half-life in the midst of several dominant precursors. Consequently, by splitting group 2 into two groups and group 6 into two groups, we obtain a total of 8 delayed neutron groups in the new model.

There was some concern expressed by several of the participants at the Obninsk workshop that by increasing the number of delayed neutron groups from 6 to 8, the quality of the LSF may be diminished because of the loss of some degrees of freedom in the LSF (i.e., 12 free parameters in an unconstrained 6-group fit vs. 8 free parameters in a constrained 8-group fit). Keepin (1965) and several other experimenters have previously argued that a LSF in which all variables are free parameters in the LSF is the most appropriate fit since it leads to the best *mathematical* representation of the decay curve as inferred from the sum-of-the-squares of the deviations. In the early 1950s, that philosophy was very logical since it was unknown at that time which precursors were actually producing the delayed neutrons; hence, it was difficult to assign any half-life to a particular group. However, we now have a much better understanding of the physics of delayed neutrons, and many of the dominant precursors have been clearly identified and their half-lives accurately measured. Therefore, it seems more appropriate in this day and age to specify those parameters in the few-group model that are known and then least-squares fit the decay curve data to obtain those remaining parameters in the model that are unknown (i.e., the relative abundances of the various groups). If the quality of a constrained 8-group fit and an unconstrained 6-group fit turn out to be essentially identical, then it is of little consequence that the sum-of-the-squares of the deviations of the constrained fit might turn out to be slightly

Table III. Comparison of the Sum-of-the-Squares of the Deviations for an Unconstrained 6-Group Fit vs. a Constrained 8-Group Fit.

Isotope	Incident Neutron Energy (MeV)	6-Group Fit	8-Group Fit	% Diff.
$^{235}\text{U}$	0.370	0.9975	1.0026	0.51
$^{235}\text{U}$	0.624	1.0277	1.0338	0.59
$^{235}\text{U}$	0.859	1.0423	1.0485	0.59
$^{235}\text{U}$	1.059	0.9139	0.9288	1.6
$^{237}\text{Np}$	0.586	1.0158	1.0291	1.3
$^{237}\text{Np}$	1.008	1.0469	1.0594	1.2
$^{237}\text{Np}$	3.745	1.0542	1.0724	1.7
$^{237}\text{Np}$	4.196	1.0743	1.1023	2.6
$^{237}\text{Np}$	4.719	0.9478	0.9518	0.42

larger than that of the unconstrained fit. After all, the intent of any LSF is to find the best values of the unknown parameters in a *given function*, as opposed to fitting data to any arbitrary function merely to minimize the sum-of-the-squares of the deviations. For example, if we had a set of ten  $(x, y)$  data points in which it was known that  $y$  varies in a linear fashion with  $x$  (i.e.,  $y=mx+b$ ), most people would perform a standard LSF to a linear function to determine the unknown parameters,  $m$  and  $b$ , and would accept the sum-of-the-squares of the deviations obtained during the fit. If the intent of the LSF, however, was to minimize the sum-of-the-squares of deviations using any arbitrary function, we could simply ignore the linear relationship between  $x$  and  $y$  and fit the ten data points to a 9<sup>th</sup>-order polynomial. The 9<sup>th</sup>-order LSF would, of course, yield a perfect fit (i.e., a zero for the sum-of-the-squares of the deviations), but the resulting model would be a significant departure from the expectation of a linear relationship.

Although this example is a bit contrived, it does nevertheless demonstrate our point. When applying this logic to the delayed neutron model, we feel that it is more appropriate to perform a least-squares fit to the sum of exponentials in which the half-lives are specified to known values rather than allowing all parameters in the model to be considered unknowns. In this way, there is no chance of cross-correlation effects and/or poor counting statistics leading to half-lives that vary from isotope-to-isotope and/or as a function of incident neutron energy merely for the sake of finding a slightly smaller sum-of-the-squares of the deviations. In the final judgement, if the constrained LSF yields the same quality of fit as the unconstrained LSF, then the constrained LSF would clearly be more advantageous from a reactor applications standpoint and from a theoretical standpoint since it would provide the consistency to the delayed neutron models of all fissionable isotopes in accordance with our current theories.

To test the conjecture that a constrained 8-group fit will yield the same quality of fit as an unconstrained 6-group model, Piksaikin (1999) analyzed the experimental data for the fast fission of  $^{235}\text{U}$  and

Table II. Variation of Group-6 Half-Life  
of Current 6-Group Delayed Neutron Model

Isotope	Half-Life (s)
$^{232}\text{Th}$	0.211
$^{231}\text{Pa}$	1.900
$^{233}\text{U}$	0.525
$^{235}\text{U}$ (thermal)	0.230
$^{235}\text{U}$ (fast)	0.185
$^{235}\text{U}$ (14 MeV)	0.160
$^{238}\text{U}$ (fast)	0.062
$^{238}\text{U}$ (14 MeV)	0.210
$^{237}\text{Np}$	0.195
$^{238}\text{Pu}$	0.428
$^{239}\text{Pu}$ (thermal)	0.257
$^{239}\text{Pu}$ (fast)	0.189
$^{240}\text{Pu}$ (fast)	0.172

$^{237}\text{Np}$ . The aggregate decay curves for these two isotopes were fit to an 8-group model using a fixed set of half-lives; the same data was also fit to an unconstrained 6-group model. The sum-of-the-squares of the deviations for these two fits are compared in Table III. Although the sum-of-the-squares in the unconstrained 6-group fit was always smaller, the sum-of-the-squares obtained during the constrained 8-group fit was only slightly higher, indicating that the two fits were essentially the same. Furthermore, when the two fits were plotted on the same figure, it was impossible to discern any differences between the two curves. Because the quality of the unconstrained 6-group model and the constrained 8-group model were, for all intents and purposes, the same for all nine sets of data shown in Table III, we surmise that the aggregate decay curves can be just as well represented by either fit. However, it should be pointed out that in the case of the unconstrained 6-group fits, the half-lives of group 1 converged to values that ranged between 53.23 and 56.37 s; hence, these 6-group results are expected to have a biased negative reactivity scale. On the other hand, the 8-group model forced the first 3 groups to correspond to the known dominant precursor half-lives, which is expected to yield a much more accurate reactivity scale than the comparable 6-group model.

Assuming similar results can be obtained for all fissionable isotopes and all incident neutron energies, we tentatively speculate that it is possible to formulate a complete set of delayed neutron parameters based on one consistent set of half-lives that will predict a more accurate reactivity scale. Unfortunately, most of the

original decay curve data for the 245 sets of parameters reported in the literature (Spriggs and Campbell, 1999) have never been published. Consequently, it would appear that we must start from ground zero to develop a new 8-group delayed neutron model (or higher-order model) based on a consistent set of half-lives. The decay curve data for all of the various fissionable isotopes and several incident neutron energies would have to be re-measured and then least-squares fit to a constrained 8-group model. These new measurements would, of course, require a great deal of time and effort by the international community, but must be eventually accomplished if known deficiencies in our current 6-group models are to be corrected.

In the interim, we have developed a technique that allows us to expand an existing delayed neutron model into an equivalent 8-group model based on a specified set of half-lives. Although this equivalent 8-group model is not considered to be as accurate as an original 8-group LSF, it does, nevertheless, correct some of the known deficiencies in our current 6-group models and, as such, should improve our current calculational capabilities.

### EXPANSION TECHNIQUE

The expansion technique is based on a least-squares fitting scheme in which simulated time-dependent behavior of a hypothetical system containing a particular isotope is least-squares fit to a higher-order delayed neutron model. In this new LSF, relative power (or flux) is the dependent variable, and both time and asymptotic inverse period are two independent variables. A detailed description of the least-squares fitting scheme follows.

Using a given set of experimentally-measured delayed neutron parameters, a series of 20 transients corresponding to step inputs of reactivity ranging from \$0.01 to \$0.95 is generated using the exact solution of the point-reactor kinetic equations. The simulated transient data is generated using the function,

$$N = \sum_{j=1}^{n+1} A_j \exp(\omega_j t) \quad , \quad (1)$$

where  $N$  is the relative neutron power,  $A_j$  is the amplitude of the  $j^{th}$  term,  $\omega_j$  is the  $j^{th}$  root of the inhour equation calculated using the *original* delayed neutron parameters, and  $n$  is the number of delayed neutron groups in the *original* model. The amplitudes,  $A_j$ , in this equation are related to the relative abundance,  $a_i$ , and decay constant of each group,  $\lambda_i$ , and to the system reactivity,  $\rho_s$ , as follows.

$$A_j = \frac{\rho_s}{\left[ \frac{\Lambda}{\beta} + \sum_{i=1}^n \frac{a_i \lambda_i}{(\omega_j + \lambda_i)^2} \right] \omega_j} \quad , \quad (2)$$

where  $\Lambda$  is the neutron generation time and  $\beta$  is the effective delayed neutron fraction. Each transient is simulated for the period of time required for the power to increase by approximately two orders of magnitude (see Fig. 1). The relative power is evaluated at one hundred points equally spaced in time.

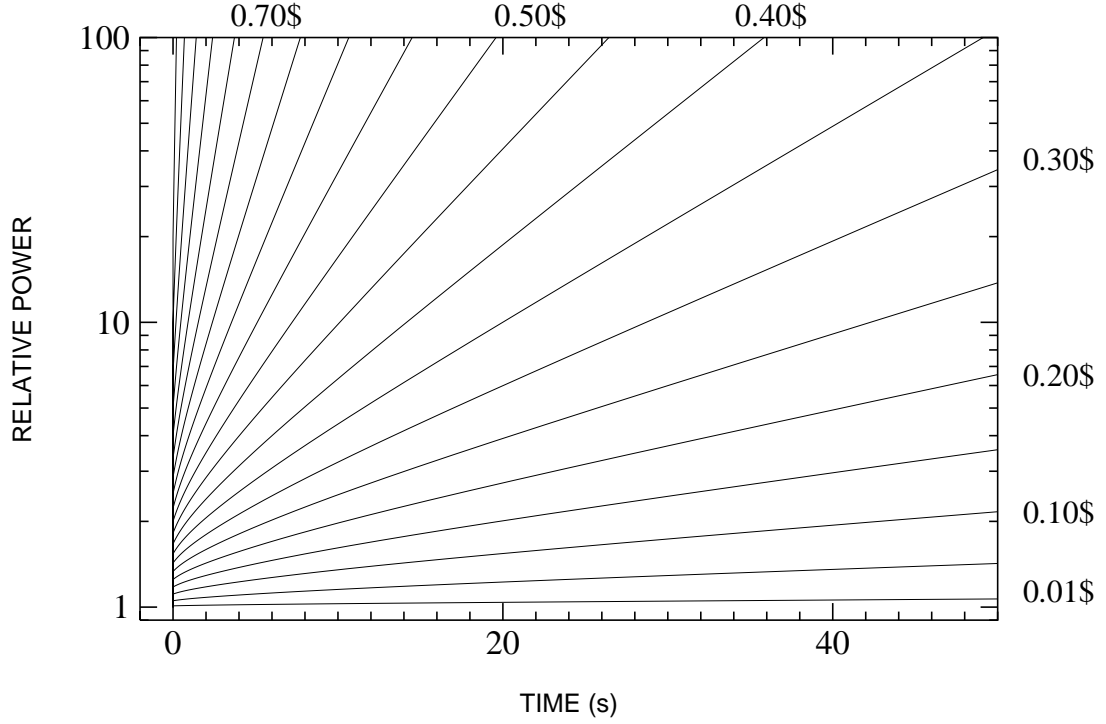


Fig. 1. Simulation of reactor power for a given few-group delayed neutron model. These data are LSF to a higher order delayed neutron model using a fixed set of half-lives.

The second independent variable—the asymptotic inverse period—is obtained by solving for the roots of the inhour equation for each reactivity used in the simulation. To ensure that the roots of the inhour equation are not significantly dependent on the behavior of prompt neutrons, the neutron generation time used in these simulations is set equal to  $10^{-12}$  s.

After collating the simulated data of relative power vs. time vs. asymptotic inverse period for a particular set of delayed neutron parameters into a single data file, these data are then simultaneously least-squares fit to a function similar to Eq. (1). This function, however, uses a higher-order delayed neutron model (i.e.,  $m$  delayed neutron groups rather than  $n$  groups). In this higher-order model, the half-lives are fixed at specified values and the amplitudes are defined by the expression

$$A_j = \frac{\omega_a \frac{\Lambda}{\beta} + \sum_{i=1}^m \frac{a_i \omega_a}{(\omega_a + \lambda_i)}}{\left[ \frac{\Lambda}{\beta} + \sum_{i=1}^m \frac{a_i \lambda_i}{(\omega_j + \lambda_i)^2} \right] \omega_j}, \quad (3)$$

Table IV. LANL's Half-Lives for 8-Group Model

Group	Precursor	Half-life (s)
1	$^{87}\text{Br}$	55.6
2	$^{137}\text{I}$	24.5
3	$^{88}\text{Br}$	16.3
4	$^{89}\text{Br}$	4.35
5	$^{90}\text{Br}$	1.91
6	$^{98}\text{Y}$	0.548
7	$^{95}\text{Rb}$	0.378
8	$^{96}\text{Rb}$	0.203

where  $\omega_a$  is the asymptotic inverse period corresponding to a particular reactivity as inferred from the original delayed neutron model. Inherent to this least-squares fitting algorithm, the following quantities are naturally conserved: 1) the time-dependent behavior of the hypothetical system as predicted using the original delayed neutron group parameters (as shown in Fig. 1), 2) the reactivity scale for *positive* periods, and 3) the mean delayed neutron half-life.

### CHOICE OF HALF-LIVES

As decided at the workshop in Obninsk, Russia, a first attempt to formulate a new delayed neutron model would be based on 8 groups of delayed neutrons. Subsequent to that workshop, two different sets of 8-group half-lives were proposed and have now been studied. The first set was based on work performed at LANL by Campbell and Spriggs (1998) in which the theoretical delayed neutron yields for 28 different fissionable isotopes were analyzed. A common set of dominant precursors in each half-live regime was identified. These precursors are listed in Table IV.

A second set of half-lives was suggested by Piksaikin (1998) based on an abundance-weighted average of three or four dominant precursors contributing to each group (with the exceptions of groups 1, 2, and 3, which correspond directly to  $^{87}\text{Br}$ ,  $^{137}\text{I}$ , and  $^{88}\text{Br}$ , respectively). These values are presented in Table V.

When comparing the results of the expansion process using the two different sets of half-lives, it was shown that Piksaikin's half-lives produced consistently lower sum-of-the-squares of the deviations during the refits. Hence, it was decided to adopt Piksaikin's half-lives as the basis for this initial 8-group study.

### UNCERTAINTY ANALYSIS

In addition to conserving the aforementioned quantities, we also wanted to conserve the uncertainty of the original reactivity scale. However, because the simulated data used in the fit had no random statistical



Table V. Piksaikin's Half-Lives for 8-Group Model

Group	Precursor	Half-life (s)	Abundance	Group Average Half-Lives (s)
1	Br-87	55.6	0.033	55.6
2	I-137	24.5	0.178	24.5
3	Br-88	16.3	0.111	16.3
4	I-138 Rb-93 Br-89	6.46 5.93 4.38	0.046 0.024 0.101	5.21
5	Rb-94 I-139 As-85 Y-98m	2.76 2.30 2.08 2.00	0.162 0.046 0.107 0.088	2.37
6	Kr-93 Cs-144 I-140	1.29 1.00 0.86	0.0048 0.0070 0.0052	1.04
7	Br-91 Rb-95	0.542 0.384	0.017 0.049	0.424
8	Rb-96 Rb-97	0.203 0.170	0.017 0.0052	0.195

fluctuations, the uncertainties of the relative abundances obtained during the expansion turned out to be unrealistically small. To obtain more realistic estimates of the uncertainties for each of the abundances obtained during the expansion, another least-squares fit was performed. In this LSF, the variance of the reactivity corresponding to the original delayed neutron model was estimated using the standard error-propagation equation.

$$\sigma_{\rho}^2 = \sum_i \left( \frac{\partial \rho}{\partial a_i} \right)^2 \sigma_{a_i}^2 + \sum_i \left( \frac{\partial \rho}{\partial \lambda_i} \right)^2 \sigma_{\lambda_i}^2 \quad \text{for } i=1,n \quad (4)$$

where

$$\frac{\partial \rho}{\partial a_i} = \frac{\omega}{(\omega + \lambda_i)} \quad , \quad (5)$$

and

$$\frac{\partial \rho}{\partial \lambda_i} = \frac{(-1)a_i \omega}{(\omega + \lambda_i)^2} \quad (6)$$

The uncertainties of the abundances and the decay constants,  $\sigma_{a_i}$  and  $\sigma_{\lambda_i}$ , are those originally quoted by the experimenter.

Note that in the above formulation, the uncertainties associated with both the abundances and the decay constants are included in the variance of the reactivity. However, in the 8-group model, it is assumed that there is no uncertainty associated with the decay constants (since they are fixed at specified values). As such, all of the uncertainty in reactivity in the expanded model is lumped into the uncertainty of the group abundances. Hence, the function that is LSF is

$$\sigma_{\rho}^2 = \sum_j \left( \frac{\partial \rho}{\partial a_j} \right)^2 \sigma_{a_j}^2 \quad \text{for } j=1,m. \quad (7)$$

In Fig. 2, we show an example of an 8-group fit to the uncertainty of the reactivity corresponding to Keepin's 6-group model for thermal fission of  $^{235}\text{U}$ .

In general, a fit of the original uncertainty values as a function of reactivity seems to maintain the *overall* uncertainty in the reactivity scale. It should be noted, however, that the uncertainty of the individual abundances in the LSF may not be physically meaningful. Undoubtedly, if the original decay-curve data were to be fit to an 8-group model, the uncertainty of each group abundance would probably be different from that obtained during this refit. Notwithstanding this potential difference, we nevertheless conserve the overall uncertainty of the reactivity scale as a function of reactivity.

## VALIDATION OF EXPANSION PROCESS

The expansion process was tested using experimental data for  $^{237}\text{Np}$ . Two least-squares fits were performed on these data. The first fit was performed using an unconstrained 6-group model (i.e., 12 free parameters), and the second fit was performed using a constrained 8-group model (i.e., 8 free parameters). The expansion technique was then applied to the 6-group model to obtain its equivalent 8-group model. A comparison of the relative abundances of the original 8-group fit and the expanded 8-group fit is shown in Table VI. As can be seen, the relative abundances are in general agreement. However, there are some noticeable differences, particularly in the short-lived groups. These differences are somewhat expected since the original 6-group model and the original 8-group model are not truly equivalent models. Although both fits provide an adequate representation of the delayed neutron decay curve, they are still different mathematical functions. Hence, the equivalent 8-group model obtained from expanding the original 6-group model would not be expected to exactly match the original 8-group model. Nevertheless, the two 8-group models should be similar providing the original 6-group fit and the original 8-group fit are comparable representations of the data. In this particular case, they appear to be fairly close.

It can also be seen from Table VI that the uncertainty of the group abundances is higher for the

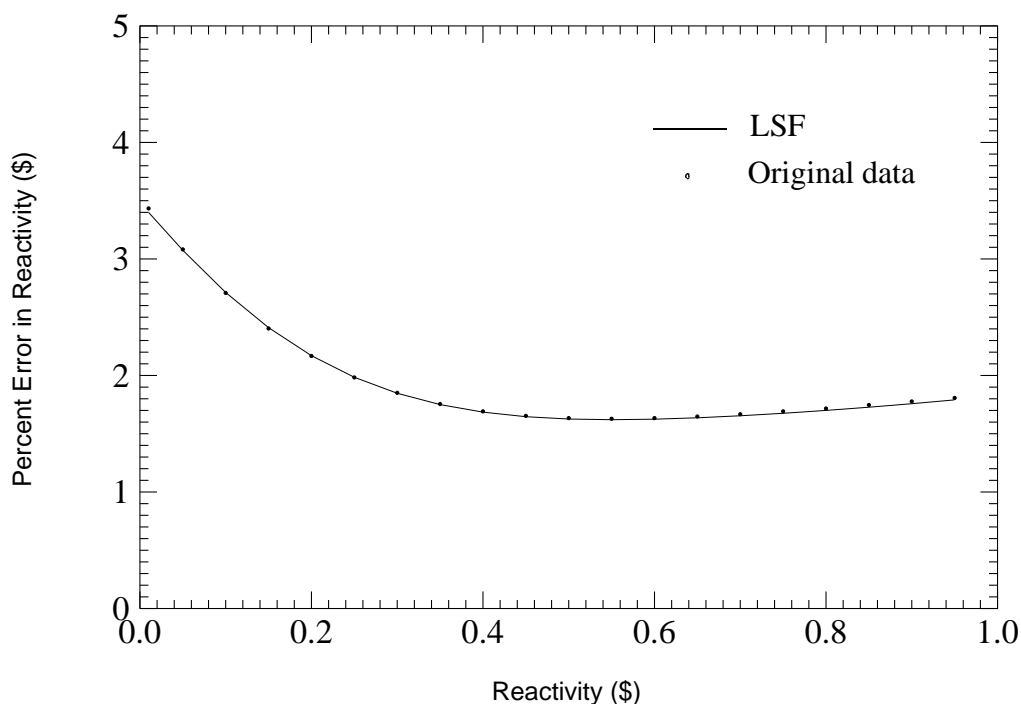


Fig. 2. Percent uncertainty ( $1\sigma$ ) in reactivity as a function of reactivity (\$) for Keepin's 6-group model of thermal fission of  $^{235}\text{U}$ .

expanded model. This increase occurs because the uncertainties of the relative abundances in the expanded 8-group model reflect the uncertainties of both the relative abundances and the decay constants obtained during the original 6-group fit. Since the uncertainty of the reactivity scale for this particular set of data was smaller for the original 8-group model than for the original 6-group fit, the differences in the uncertainties of the relative abundances between the expanded 8-group model and the original 8-group model are justifiable.

## EXPANSION RESULTS

Over the past 50 years, 245 sets of delayed neutron parameters for 20 different isotopes have been published in the open literature. The results of the expansion process for these 245 sets of parameters are listed in another reference (Spriggs, Campbell, and Piksaikin, 1999). It should be noted that when the expansions were performed, there were many instances when the number of groups in the final expanded model corresponded to less than 8 groups. This failure to expand to a full 8-group set occurred for those delayed neutron sets in which the experimenters were unable to resolve the very short-lived groups normally represented in the 6-group models. Without the presence of the short-lived groups in the original model, there was little hope of obtaining convergence of any short-lived group in the expanded model. As a general rule, it was found that convergence of the full 8-group model could not be obtained if the half-life of the shortest-lived group in the expanded model was significantly shorter than the half-life of the shortest-lived group in the original model. Consequently, groups with a half-life significantly smaller than the shortest measured half-

Table VI. Comparison of Expanded 8-Group Model  
to Original 8-Group Model

Group	Half-Life (s)	Relative Abundances: 8-Group Fit	Relative Abundances: Expanded 8-Group Model
1	55.7	$.030 \pm 0.10\text{E-}02$	$.030 \pm 0.16\text{E-}02$
2	24.5	$.172 \pm 0.40\text{E-}02$	$.178 \pm 0.56\text{E-}02$
3	16.3	$.103 \pm 0.30\text{E-}02$	$.097 \pm 0.56\text{E-}02$
4	5.21	$.163 \pm 0.40\text{E-}02$	$.175 \pm 0.83\text{E-}02$
5	2.37	$.364 \pm 0.60\text{E-}02$	$.337 \pm 0.67\text{E-}02$
6	1.04	$.018 \pm 0.10\text{E-}02$	$.051 \pm 0.35\text{E-}02$
7	.424	$.128 \pm 0.60\text{E-}02$	$.124 \pm 0.62\text{E-}02$
8	.195	$.022 \pm 0.10\text{E-}02$	$.010 \pm 0.19\text{E-}03$

life were not used in the expanded model. For example, if the shortest-lived group in the original model corresponded to 0.5 s, then the expanded model used only 7 groups since the half-life of group 7 is 0.424 s (see Table V). In this example, group 8 was not included in the expansion since its half-life is 0.195 s, which is too small relative to the 0.5 s half-life of the original model.

## SELECTION CRITERIA

After obtaining the results of the expansion process, we now come to the most difficult part of this project—the recommendation as to which 8-group model best represents the delayed neutron activity for the various isotopes. This task is made very difficult for two reasons: 1) there is considerable spread in the experimentally-measured delayed neutron activity curve for most of the isotopes; for example, Fig. 2 shows a comparison of the 16 measurements of the delayed neutron decay curve for the thermal fissioning of  $^{235}\text{U}$ , and 2) it is nearly impossible to test the recommendations for the vast majority of the isotopes using independent techniques, such as in-core reactivity measurements or time-dependent reactor behavior. Faced with these difficulties, how does one determine the *best* set of delayed neutron parameters for reactor physics application?

Although we have tried to be very systematic in our choices, ultimately it comes down to a somewhat subjective opinion as to which delayed neutron sets seem best. Notwithstanding the inherently subjective nature of our choices, we have, nevertheless, tried to follow some general selection criteria to aid us in this effort. These criteria are described below.

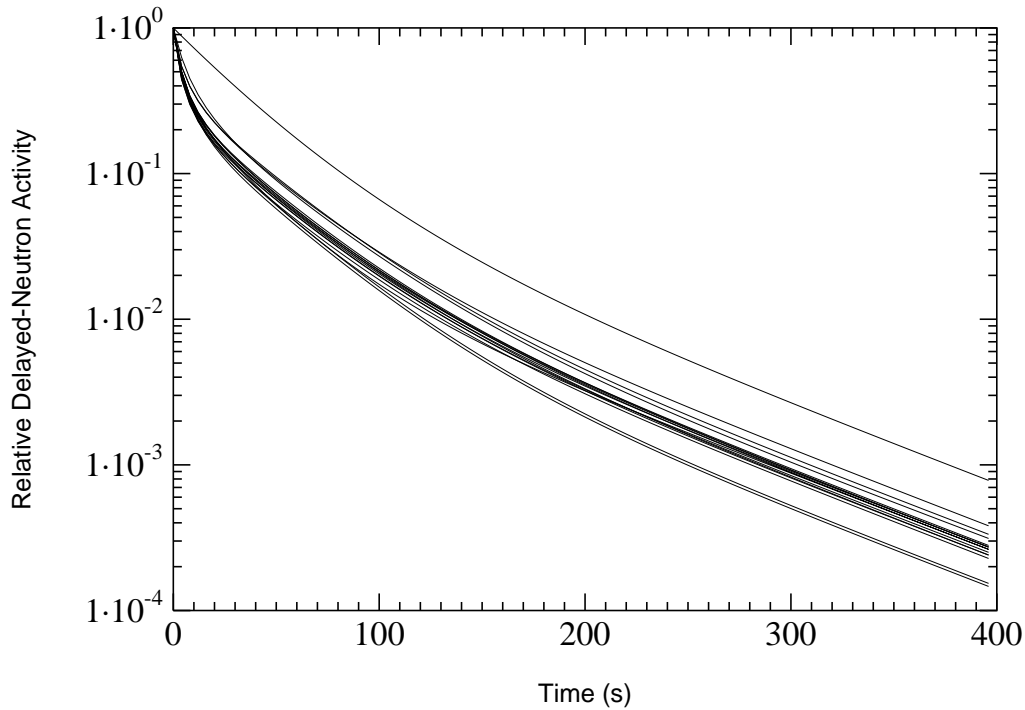


Fig. 3. Comparison of experimentally-measured delayed neutron decay curves for the thermal fission of  $^{235}\text{U}$ .

### Sample Transfer Time

One of the most important parameters in a delayed neutron decay curve measurement is the sample transfer time. Because a significant number of delayed neutrons decay within just a few milliseconds following irradiation, if the sample transfer time is too slow, then it becomes very difficult to observe these short-lived precursors. This failure to observe the short-lived precursors, in turn, makes it nearly impossible to obtain a complete picture of the delayed neutron activity curve as a function of time, which, in turn, causes an overestimate of the relative abundances of the delayed neutron groups that are observed. Obviously, those experiments in which the sample transfer times were relatively long would show the least amount of decrease during the initial portion of the decay curve since the short-lived precursors were not properly accounted for in the fit. Consequently, one would expect a strong correlation of the relative position of the various decay curves as a function of the sample transfer time. That is, the experiments with the longest sample transfer times should yield a higher delayed neutron activity at a given point in time than those experiments with relatively short sample transfer times. However, this was not always the case. As noted from the comparisons of the decay curves for the various isotopes (Spriggs and Campbell, 1999), several of the long-transfer-time experiments fall well below the relatively short-transfer-time experiments. Although we cannot explain this apparent discrepancy at this time, we have, nevertheless, placed great emphasis on the sample transfer time and have given more weight to those measurements in which the sample transfer times were short.

## Graphical vs. Least-Squares-Fit

Analyses of most of the early delayed neutron measurements were performed using a graphical stripping technique. In this stripping technique, the experimenter starts at the tail-end of the decay curve (plotted in the semi-log plane) and fits the last few points (using a straight-edge) to a simple exponential function to obtain the longest-lived precursor group. This exponential function is then subtracted from the remaining data to obtain another curve that does not contain this long-lived term. The process is then repeated to obtain the next longest-lived group, and so on and so forth. Needless to say, this graphical stripping technique is very subjective and, depending on who is drawing the straight lines, the decay constants and the group abundances can vary significantly for the same data set.

With the advent of computers in the mid 1950s, the graphical stripping technique was eventually replaced with the automated processing of the experimental data by way of a least-squares fit (LSF) analysis. Not only did the LSF remove some of the subjectivity associated with the graphical stripping technique, but it also allowed for a more realistic estimate of the uncertainties of each of the parameters obtained during the fit. Consequently, when faced with a choice of using results obtained from a LSF or a graphical stripping technique, we usually opted for the LSF.

## Standard Deviation of Parameters

In most cases, the sum-of-the-squares of the deviations of the LSFs were not reported in the literature. On the other hand, the standard deviation of each of the parameters obtained during the fits were usually quoted, with the notable exception of those results obtained using the graphical stripping technique. So, to obtain a measure of the quality of the fit, we calculated the uncertainty of the area under the delayed neutron decay curve over the time regime of 10 s to 300 s. We chose this particular time regime because of the wide variation in the sample transfer time. It was felt that even if an experimenter was unable to transfer the sample as fast as other experimenters, that did not necessarily preclude the experimenter from obtaining a high precision measurement of the portion of the decay curve that was measured. Since most transfer times were 5 s or less, we chose 10 s to be the initial starting point of the area integral and the uncertainty calculation. The final point of the area integral and its uncertainty was chosen to be 300 s since most experimenters were not able to go much beyond that point in time before reaching background.

We then arbitrarily defined a *weight*,  $W_i$ , associated with each measurement as the reciprocal of the product of the sample transfer time and the variance of the area from 10 s to 300 s.

$$W_i = \frac{1}{\tau_i \sigma_i^2} \quad , \quad (8)$$

where  $\tau_i$  is the sample transfer time and  $\sigma_i^2$  is the variance of the area under the decay curve from 10 s to 300 s. Consequently, those experiments that had a short sample transfer time (which we deem to be very important) and a small variance (which we also deem to be very important) received the highest weight. This weight was then used to determine if any one experiment was significantly better than other experiments for a given isotope. A summary of these results is given in the previously mentioned reference by Spriggs, Campbell, and Piksaikin (1999).

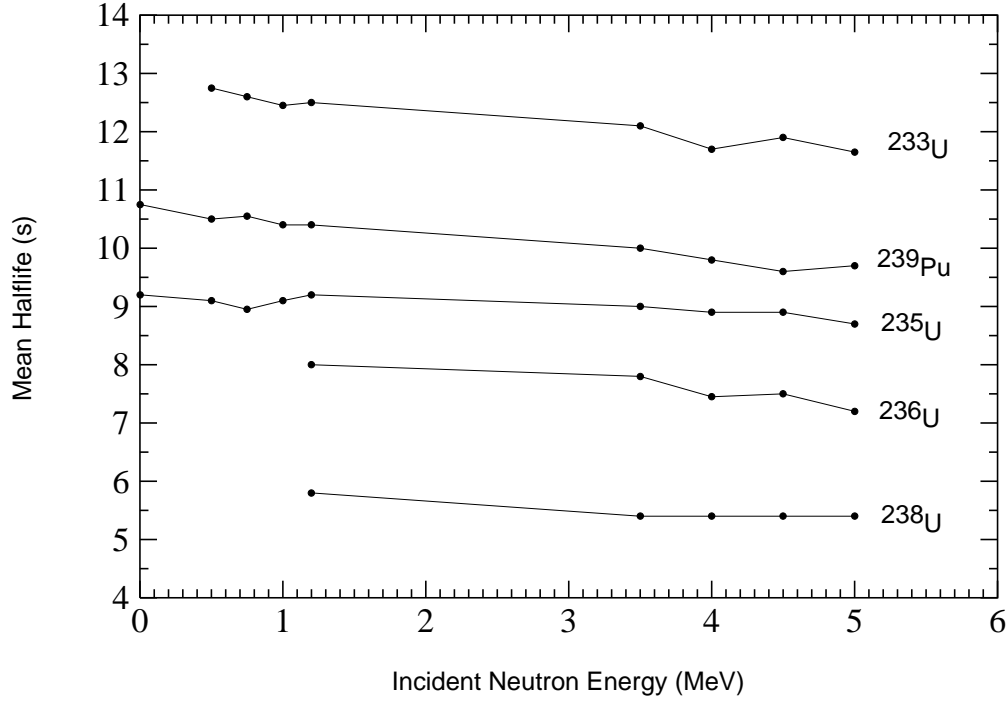


Fig. 4. Preliminary experimental results (Piksaikin, 1999) showing the abundance-weighted delayed neutron half-life as a function of incident neutron energy. (Note, owing to the preliminary nature of these data, they have not yet been included in the data base considered in this report.)

### Consistency of Mean Half-Life

Another parameter that was deemed to be of great importance was the abundance-weighted mean half-life of the delayed neutrons defined as

$$\bar{t} = \sum_i a_i t_i \quad , \quad (9)$$

where  $a_i$  is the relative abundance of the  $i^{th}$  group and  $t_i$  is the half-life of the  $i^{th}$  group. It has been experimentally observed that the mean half-life decreases as a function of the incident neutron energy. This is clearly demonstrated by the recent results obtained by Piksaikin (1999) (see Fig. 4). Therefore, when choosing a set of delayed neutron parameters for thermal, fast, and high energy fissioning of a particular isotope, we wanted the mean half-lives to decrease somewhat with increasing neutron energy.

### Qualitative Agreement with Systematic Models

As shown by Piksaikin and Isaev (1999), the mean delayed neutron half-life corresponding to a given set of delayed neutron parameters is an important variable in delayed neutron systematics. It was found that the mean half-life increases exponentially with the variable  $P$  defined as

$$P = -\frac{(A_c - 3Z)A_c}{Z}, \quad (10)$$

where  $A_c$  and  $Z$  are the mass number and atomic number of the compound nucleus, respectively. Using the mean half-lives of several of the commonly accepted measured delayed neutron sets for thorium, uranium, plutonium, and americium, Piksaikin and Isaev (1998) developed several empirical correlations of mean half-life vs.  $P$  for those particular isotopes. Consequently, if you accept the empirical correlations to be true, then for a particular thorium, uranium, plutonium, or americium 8-group model to be acceptable, one would expect qualitative agreement of the mean half-life of the 8-group set with its respective correlation.

### Quality of 8-Group Expansion

And finally, the quality of the 8-group expansion was considered when selecting an 8-group representation for a particular isotope. In many instances, the original 4-, 5-, or 6-group delayed neutron results would not expand to a full 8-group model. When this occurred, preference was given to those models that did expand to a full 8 groups. But, that is not to say that all 8-group models were chosen at the exclusion of all 7-group models. On occasion, a 4-, 5-, or 6- group model would expand to a full 8-group model, but the relative abundances for several of the interior groups would converge to a very small number (essentially 0.0). Hence, a 7-group model in which the relative abundances were well behaved would be chosen over an 8-group model containing zero relative abundances.

## RECOMMENDED 8-GROUP SETS

Using the selection criteria described in the previous section, we have selected an 8-group model (or, in some cases, a 6- or 7-group model) that we think best satisfies all or most of the selection criteria. In addition, we have also tried to maintain the traditional distinction of a different set of delayed neutron parameters for thermal-, fast-, and high-energy fissioning. We have not, however, make any recommendations for incident neutron energies in the transitional regions. Most of the measurements in the transitional regions were performed by Maksyutenko (see Spriggs and Campbell (1999) for a complete listing of these measurements) using sample transfer times of 5 s. As a result of this relatively long sample transfer time, Maksyutenko was unable to resolve the short-lived precursors. This failure to resolve the short-lived precursors, in turn, resulted in mean half-lives that were much larger than the half-lives corresponding to the thermal-, fast-, or high-energy fissioning of the same isotope. For example, in the case of  $^{235}\text{U}$ , the mean half-lives for thermal-, fast-, and high-energy fissioning are 9.03, 9.10, and 8.97 s, respectively. In contrast, the mean half-life in the transitional region is seen to vary from 9.89 to 14.2 s. Hence, the absence of the short-lived group(s) has resulted in too high of a value. Consequently, we feel that the delayed neutron parameters measured in this energy region are biased on the high side and, as such, do not warrant inclusion in our recommendations. A complete set of our recommendations are given in Table VII.

As a check on our recommendations, we have also included plots of the 8-group mean half-lives of the thorium, uranium, plutonium, and americium isotopes as a function of  $P$  (see Fig. 5 and Fig. 6). These plots seem to confirm that the 8-group models that we have chosen for the isotopes of those four elements are consistent with current theory.



Table VII. Recommended 8-Group Delayed Neutron Parameters

Isotope/Group	Half-Life (s)	Relative Abundances $\pm 1\sigma$		
		Thermal	Fast	High
$^{229}\text{Th}$		#1 <sup>a</sup> Gudkov (1989)		
1	55.6	.113 $\pm$ 10.%		
2	24.5	.250 $\pm$ 9.%		
3	16.3	.124 $\pm$ 11.%		
4	5.21	.242 $\pm$ 8.%		
5	2.37	.178 $\pm$ 9.%		
6	1.04	.071 $\pm$ 20.%		
7	.424	.022 $\pm$ 42.%		
8	.195			
	Mean $T_{1/2} =$	16.19 $\pm$ 5.%		
$^{232}\text{Th}$			#5 Keepin (1957)	#28 Maksyu. (1958)
1	55.6		.033 $\pm$ 8.%	.037 $\pm$ 11.%
2	24.5		.073 $\pm$ 7.%	.074 $\pm$ 10.%
3	16.3		.093 $\pm$ 2.%	.098 $\pm$ 15.%
4	5.21		.136 $\pm$ 18.%	.209 $\pm$ 8.%
5	2.37		.381 $\pm$ 2.%	.262 $\pm$ 15.%
6	1.04		.140 $\pm$ 6.%	.219 $\pm$ 11.%
7	.424		.114 $\pm$ 11.%	.101 $\pm$ 15.%
8	.195		.030 $\pm$ 3.%	
	Mean $T_{1/2} =$		6.95 $\pm$ 3.%	7.45 $\pm$ 5.%

Table VII. Recommended 8-Group Delayed Neutron Parameters

Isotope/Group	Half-Life (s)	Relative Abundances $\pm 1\sigma$		
		Thermal	Fast	High
$^{231}\text{Pa}$			#32 Anoussis (1973)	#33 Brown (1971)
1	55.6		$.115 \pm 1\%$	$.126 \pm 10\%$
2	24.5		$.099 \pm 2\%$	$.068 \pm 24\%$
3	16.3		$.228 \pm 3\%$	$.232 \pm 9\%$
4	5.21		$.181 \pm 14\%$	$.205 \pm 14\%$
5	2.37		$.353 \pm 8\%$	$.341 \pm 9\%$
6	1.04		$.024 \pm 42\%$	$.028 \pm 43\%$
7	.424			
8	.195			
	Mean $T_{1/2} =$		$14.34 \pm 1\%$	$14.36 \pm 6\%$
$^{232}\text{U}$		#34 Waldo (1981)		
1	55.6	$.109 \pm 8\%$		
2	24.5	$.144 \pm 10\%$		
3	16.3	$.178 \pm 11\%$		
4	5.21	$.218 \pm 15\%$		
5	2.37	$.270 \pm 2\%$		
6	1.04	$.076 \pm 63\%$		
7	.424	$.005 \pm 1600\%$		
8	.195			
	Mean $T_{1/2} =$	$14.35 \pm 5\%$		

Table VII. Recommended 8-Group Delayed Neutron Parameters

Isotope/Group	Half-Life (s)	Relative Abundances $\pm 1\sigma$		
		Thermal	Fast	High
$^{233}\text{U}$		#37 Keepin (1957)	#42 Maksyu. (1967)	#51 East (1970)
1	55.6	.080 $\pm$ 5.%	.080 $\pm$ 8.%	.093 $\pm$ 2.%
2	24.5	.167 $\pm$ 2.%	.157 $\pm$ 2.%	.078 $\pm$ 2.%
3	16.3	.150 $\pm$ 2.%	.135 $\pm$ 2.%	.140 $\pm$ 2.%
4	5.21	.200 $\pm$ 20.%	.209 $\pm$ 18.%	.204 $\pm$ 9.%
5	2.37	.298 $\pm$ 7.%	.308 $\pm$ 2.%	.330 $\pm$ 2.%
6	1.04	.039 $\pm$ 2.%	.037 $\pm$ 2.%	.058 $\pm$ 16.%
7	.424	.056 $\pm$ 45.%	.062 $\pm$ 14.%	.072 $\pm$ 2.%
8	.195	.010 $\pm$ 2.%	.012 $\pm$ 92.%	.025 $\pm$ 6.%
	Mean $T_{1/2} =$	12.80 $\pm$ 2.%	12.38 $\pm$ 3.%	11.30 $\pm$ 1.%
$^{235}\text{U}$		#68 Keepin (1957)	#88 Piksaikin (1997)	#108 East (1970)
1	55.6	.033 $\pm$ 13.%	.034 $\pm$ 2.%	.052 $\pm$ 2.%
2	24.5	.154 $\pm$ 4.%	.150 $\pm$ 2.%	.099 $\pm$ 2.%
3	16.3	.091 $\pm$ 10.%	.099 $\pm$ 3.%	.107 $\pm$ 4.%
4	5.21	.197 $\pm$ 12.%	.200 $\pm$ 2.%	.185 $\pm$ 12.%
5	2.37	.331 $\pm$ 2.%	.312 $\pm$ 2.%	.346 $\pm$ 2.%
6	1.04	.090 $\pm$ 5.%	.093 $\pm$ 4.%	.079 $\pm$ 11.%
7	.424	.081 $\pm$ 2.%	.087 $\pm$ 5.%	.087 $\pm$ 2.%
8	.195	.023 $\pm$ 41.%	.025 $\pm$ 4.%	.045 $\pm$ 18.%
	Mean $T_{1/2} =$	9.03 $\pm$ 4.%	9.10 $\pm$ 1.%	8.97 $\pm$ 2.%

Table VII. Recommended 8-Group Delayed Neutron Parameters

Isotope/Group	Half-Life (s)	Relative Abundances $\pm 1\sigma$		
		Thermal	Fast	High
$^{236}\text{U}$			#115 Gudkov (1989)	
1	55.6		$.025 \pm 16\%$	
2	24.5		$.098 \pm 18\%$	
3	16.3		$.108 \pm 21\%$	
4	5.21		$.127 \pm 20\%$	
5	2.37		$.410 \pm 18\%$	
6	1.04		$.137 \pm 26\%$	
7	.424		$.088 \pm 19\%$	
8	.195		$.007 \pm 143\%$	
	Mean $T_{1/2} =$		$7.37 \pm 9\%$	
$^{238}\text{U}$			#118 Keepin (1957)	#148 East (1970)
1	55.6		$.008 \pm 16\%$	$.016 \pm 4\%$
2	24.5		$.104 \pm 2\%$	$.089 \pm 2\%$
3	16.3		$.038 \pm 2\%$	$.051 \pm 6\%$
4	5.21		$.137 \pm 15\%$	$.141 \pm 3\%$
5	2.37		$.294 \pm 4\%$	$.325 \pm 2\%$
6	1.04		$.198 \pm 1\%$	$.151 \pm 2\%$
7	.424		$.128 \pm 10\%$	$.121 \pm 2\%$
8	.195		$.093 \pm 4\%$	$.106 \pm 4\%$
	Mean $T_{1/2} =$		$5.30 \pm 3\%$	$5.64 \pm 1\%$

Table VII. Recommended 8-Group Delayed Neutron Parameters

Isotope/Group	Half-Life (s)	Relative Abundances $\pm 1\sigma$		
		Thermal	Fast	High
$^{237}\text{Np}$			#190 Piksaikin (1997)	
1	55.6		$.035 \pm 2\%$	
2	24.5		$.149 \pm 2\%$	
3	16.3		$.089 \pm 2\%$	
4	5.21		$.167 \pm 2\%$	
5	2.37		$.373 \pm 1\%$	
6	1.04		$.021 \pm 3\%$	
7	.424		$.141 \pm 3\%$	
8	.195		$.025 \pm 3\%$	
	Mean $T_{1/2} =$		$8.89 \pm 1\%$	
$^{238}\text{Pu}$		#195 Waldo (1981)	#196 Benedetti (1982)	
1	55.6	$.042 \pm 22\%$	$.045 \pm 18\%$	
2	24.5	$.219 \pm 12\%$	$.250 \pm 7\%$	
3	16.3	$.137 \pm 42\%$	$.052 \pm 2\%$	
4	5.21	$.134 \pm 49\%$	$.256 \pm 5\%$	
5	2.37	$.386 \pm 2\%$	$.251 \pm 14\%$	
6	1.04	$.066 \pm 152\%$	$.119 \pm 10\%$	
7	.424	$.016 \pm 1062\%$	$.027 \pm 59\%$	
8	.195			
	Mean $T_{1/2} =$	$11.62 \pm 11\%$	$11.54 \pm 6\%$	

Table VII. Recommended 8-Group Delayed Neutron Parameters

Isotope/Group	Half-Life (s)	Relative Abundances $\pm 1\sigma$		
		Thermal	Fast	High
$^{239}\text{Pu}$		#199 Keepin (1957)	#207 Besant (1977)	#214 Maksyu. (1963)
1	55.6	$.032 \pm 38\%$	$.029 \pm 7\%$	$.049 \pm 2\%$
2	24.5	$.237 \pm 14\%$	$.225 \pm 2\%$	$.145 \pm 2\%$
3	16.3	$.083 \pm 2\%$	$.095 \pm 10\%$	$.053 \pm 8\%$
4	5.21	$.182 \pm 29\%$	$.149 \pm 29\%$	$.212 \pm 3\%$
5	2.37	$.294 \pm 10\%$	$.351 \pm 2\%$	$.312 \pm 2\%$
6	1.04	$.082 \pm 2\%$	$.037 \pm 51\%$	$.121 \pm 40\%$
7	.424	$.072 \pm 43\%$	$.097 \pm 94\%$	$.108 \pm 93\%$
8	.195	$.018 \pm 2\%$	$.017 \pm 229\%$	
	Mean $T_{1/2} =$	$10.70 \pm 10\%$	$10.36 \pm 3\%$	$9.16 \pm 1\%$
$^{240}\text{Pu}$			#224 Keepin (1957)	
1	55.6		$.022 \pm 15\%$	
2	24.5		$.207 \pm 2\%$	
3	16.3		$.080 \pm 2\%$	
4	5.21		$.161 \pm 34\%$	
5	2.37		$.314 \pm 3\%$	
6	1.04		$.105 \pm 9\%$	
7	.424		$.079 \pm 22\%$	
8	.195		$.032 \pm 9\%$	
	Mean $T_{1/2} =$		$9.33 \pm 4\%$	

Table VII. Recommended 8-Group Delayed Neutron Parameters

Isotope/Group	Half-Life (s)	Relative Abundances $\pm 1\sigma$		
		Thermal	Fast	High
$^{241}\text{Pu}$		#227 Cox (1961)	#230 Gudkov (1989)	
1	55.6	$.011 \pm 30\%$	$.016 \pm 14\%$	
2	24.5	$.166 \pm 2\%$	$.175 \pm 11\%$	
3	16.3	$.094 \pm 12\%$	$.055 \pm 22\%$	
4	5.21	$.100 \pm 25\%$	$.170 \pm 11\%$	
5	2.37	$.382 \pm 11\%$	$.280 \pm 13\%$	
6	1.04	$.073 \pm 41\%$	$.166 \pm 20\%$	
7	.424	$.174 \pm 7\%$	$.113 \pm 31\%$	
8	.195		$.025 \pm 25\%$	
	Mean $T_{1/2} =$	$7.79 \pm 4\%$	$7.85 \pm 7\%$	
$^{242}\text{Pu}$			#231 Waldo (1981)	#233 East (1970)
1	55.6		$.014 \pm 2\%$	$.022 \pm 21\%$
2	24.5		$.095 \pm 54\%$	$.097 \pm 2\%$
3	16.3		$.134 \pm 11\%$	$.090 \pm 2\%$
4	5.21		$.033 \pm 61\%$	$.108 \pm 17\%$
5	2.37		$.404 \pm 2\%$	$.366 \pm 1\%$
6	1.04		$.001 \pm 6000\%$	$.111 \pm 2\%$
7	.424		$.258 \pm 18\%$	$.143 \pm 7\%$
8	.195		$.061 \pm 85\%$	$.063 \pm 10\%$
	Mean $T_{1/2} =$		$6.54 \pm 20\%$	$6.69 \pm 4\%$

Table VII. Recommended 8-Group Delayed Neutron Parameters

Isotope/Group	Half-Life (s)	Relative Abundances $\pm 1\sigma$		
		Thermal	Fast	High
$^{241}\text{Am}$		#234 Waldo (1981)	#237 Gudkov (1989)	
1	55.6	$.034 \pm 9\%$	$.039 \pm 18\%$	
2	24.5	$.238 \pm 14\%$	$.171 \pm 15\%$	
3	16.3	$.061 \pm 20\%$	$.114 \pm 16\%$	
4	5.21	$.182 \pm 18\%$	$.199 \pm 18\%$	
5	2.37	$.305 \pm 11\%$	$.258 \pm 11\%$	
6	1.04	$.106 \pm 2\%$	$.085 \pm 72\%$	
7	.424	$.038 \pm 174\%$	$.114 \pm 77\%$	
8	.195	$.036 \pm 200\%$	$.020 \pm 2\%$	
	Mean $T_{1/2} =$	$10.52 \pm 8\%$	$10.00 \pm 8\%$	
$^{242\text{m}}\text{Am}$		#238 Waldo (1981)		
1	55.6	$.021 \pm 2\%$		
2	24.5	$.245 \pm 7\%$		
3	16.3	$.060 \pm 13\%$		
4	5.21	$.205 \pm 12\%$		
5	2.37	$.261 \pm 11\%$		
6	1.04	$.179 \pm 22\%$		
7	.424	$.029 \pm 193\%$		
8	.195			
	Mean $T_{1/2} =$	$10.03 \pm 5\%$		



Table VII. Recommended 8-Group Delayed Neutron Parameters

Isotope/Group	Half-Life (s)	Relative Abundances $\pm 1\sigma$		
		Thermal	Fast	High
$^{243}\text{Am}$			#241 Charlton (1998)	
1	55.6		$.018 \pm 32\%$	
2	24.5		$.220 \pm 5\%$	
3	16.3		$.098 \pm 2\%$	
4	5.21		$.121 \pm 7\%$	
5	2.37		$.316 \pm 4\%$	
6	1.04		$.170 \pm 2\%$	
7	.424		$.043 \pm 26\%$	
8	.195		$.014 \pm 16\%$	
	Mean $T_{1/2} =$		$9.57 \pm 5\%$	
$^{245}\text{Cm}$		#242 Waldo (1981)		
1	55.6	$.016 \pm 27\%$		
2	24.5	$.269 \pm 7\%$		
3	16.3	$.045 \pm 2\%$		
4	5.21	$.204 \pm 23\%$		
5	2.37	$.255 \pm 16\%$		
6	1.04	$.178 \pm 28\%$		
7	.424	$.033 \pm 255\%$		
8	.195			
	Mean $T_{1/2} =$	$10.08 \pm 6\%$		

Table VII. Recommended 8-Group Delayed Neutron Parameters

Isotope/Group	Half-Life (s)	Relative Abundances $\pm 1\sigma$		
		Thermal	Fast	High
<sup>249</sup> Cf		#243 Waldo (1981)		
1	55.6	.024 $\pm$ 2.%		
2	24.5	.292 $\pm$ 8.%		
3	16.3	.064 $\pm$ 17.%		
4	5.21	.228 $\pm$ 12.%		
5	2.37	.265 $\pm$ 10.%		
6	1.04	.127 $\pm$ 13.%		
7	.424			
8	.195			
	Mean $T_{1/2}$ =	11.48 $\pm$ 5.%		
<sup>252</sup> Cf		#245 Chulick (1969)		
1	55.6	.014 $\pm$ 44.%		
2	24.5	.318 $\pm$ 2.%		
3	16.3	.001 $\pm$ 2400.%		
4	5.21	.209 $\pm$ 9.%		
5	2.37	.200 $\pm$ 2.%		
6	1.04	.144 $\pm$ 22.%		
7	.424	.114 $\pm$ 39.%		
8	.195			
	Mean $T_{1/2}$ =	10.35 $\pm$ 5.%		

a. The number above the recommended delayed neutron set corresponds to the data set number presented in Spriggs and Campbell (1999).

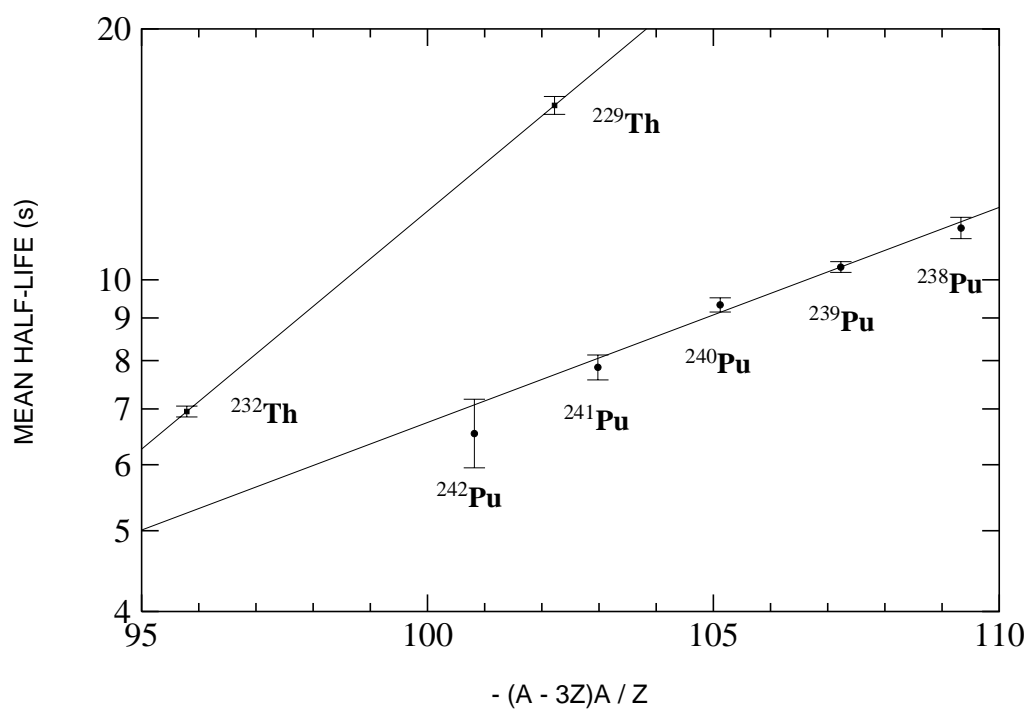


Fig. 5. Mean half-lives of thorium and plutonium isotopes as a function  $P$ .

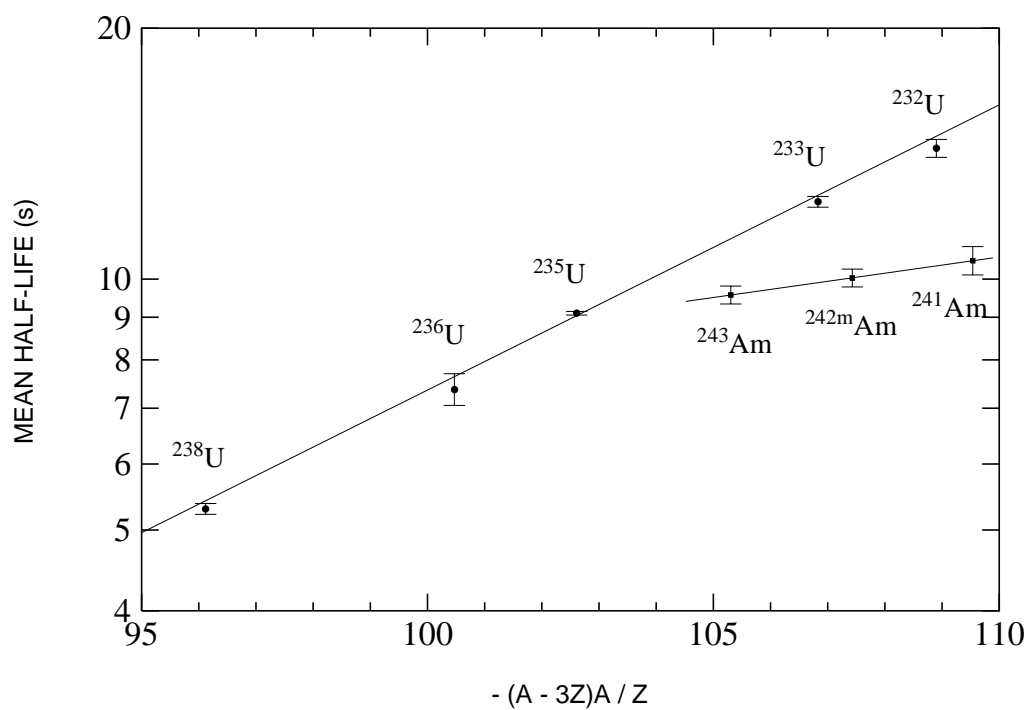


Fig. 6. Mean half-lives of uranium and americium isotopes as a function of  $P$ .

## CONCLUSIONS

In this study, we have demonstrated that it is possible to develop a higher-order delayed neutron model based on a consistent set of half-lives for all fissioning isotopes and all incident neutron energies. Although the choice of half-lives can be argued, the need for a consistent set of half-lives is still clear. A *universal* set of half-lives would not only simplify the modeling of delayed neutrons in complex reactor systems, but would also provide a more consistent basis for comparison of experimental results obtained by various experimenters, and would greatly enhance our ability to validate and/or improved our current delayed neutron models.

Unfortunately, to obtain the 8-group model, we had to revert to an expansion technique that is basically a least-squares fit of a least-squares fit. Although the expansion technique preserves the positive portion of the reactivity scale and its overall uncertainty, the mean delayed neutron half-life, and the time-dependent behavior of the system as predicted by the original delayed neutron parameters, if given a choice, we would certainly have opted to refit the original data to an 8-group model. But, in most cases, the original experimental data for the 245 sets of delayed neutron parameters included in this report are no longer available for refitting. Therefore, until new data can be collected and analyzed, the expansion process presented in this study is viewed by us as being a *temporary* solution. The expansion technique merely allows us to utilize existing 4-, 5-, and 6-group models to achieve an improved delayed neutron model that can be used immediately in reactor physics applications. Therefore, we hope that the reader views the results presented in this study as merely an *interim* step towards the development of new and improved delayed neutron models for future reactor physics applications.

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